

## Appendix

### Exponential function

$$e^x e^y = e^{x+y}$$

$$e^x / e^y = e^{x-y}$$

$$(e^x)^y = e^{xy}$$

$$e^{\ln x} = x$$

$$e^{-\ln x} = e^{\ln(1/x)} = \frac{1}{x}$$

### Trigonometric functions

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan x = \frac{\sin x}{\cos x}, \cot x = \frac{\cos x}{\sin x}, \sec x = \frac{1}{\cos x}, \csc x = \frac{1}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \sin y = \frac{1}{2} \{-\cos(x+y) + \cos(x-y)\}$$

$$\cos x \cos y = \frac{1}{2} \{\cos(x+y) + \cos(x-y)\}$$

$$\sin x \cos y = \frac{1}{2} \{\sin(x+y) + \sin(x-y)\}$$

$$\sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \sin(x + \phi)$$

where

$$\cos \phi = \frac{a}{\sqrt{a^2 + b^2}}, \quad \sin \phi = \frac{b}{\sqrt{a^2 + b^2}}$$

$$e^{\pm ix} = \cos x \pm i \sin x \quad (\text{Euler's identities})$$

### Differentiations

$$(cu)' = cu' \quad (c \text{ constant})$$

$$(u+v)' = u' + v'$$

$$(uv)' = u'v + v'u, \quad (u/v)' = (u'v - v'u)/v^2$$

$$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx} \quad (\text{chain rule})$$

$$(x^n)' = nx^{n-1}$$

$$(e^x)' = e^x$$

$$(a^x)' = a^x \ln a$$

$$(x^x)' = x^x(1 + \ln x) \quad (x > 0, x \neq 1)$$

### Natural logarithmic function

$$\ln(xy) = \ln x + \ln y$$

$$\ln(x/y) = \ln x - \ln y$$

$$\ln(x^a) = a \ln x$$

### Logarithmic function

$$\log_a x = \frac{\ln x}{\ln a}$$

### Hyperbolic functions

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{\sinh x}{\cosh x}, \quad \coth x = \frac{\cosh x}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

### Gamma function

$$\Gamma(a) = \int_0^\infty e^{-t} t^{a-1} dt \quad (a > 0)$$

$$\Gamma(a+1) = a\Gamma(a), \quad \Gamma(k+1) = k! \quad (k = 0, 1, \dots) \quad (\text{Factorial})$$

$$\Gamma(1/2) = \sqrt{\pi}$$

### Inverse hyperbolic functions (principal values)

$$\operatorname{arcsinh} x = \ln(x + \sqrt{x^2 + 1}) \quad -\infty < x < \infty$$

$$\operatorname{arccosh} x = \ln(x + \sqrt{x^2 - 1}) \quad x \geq 1 \quad (\operatorname{arccosh} x > 0)$$

$$\operatorname{arctanh} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right) \quad -1 < x < 1$$

$$\operatorname{arcoth} x = \frac{1}{2} \ln \left( \frac{x+1}{x-1} \right) \quad x < -1, \quad x > 1$$

### Integrations

$$\int cu \, dx = c \int u \, dx \quad (c \text{ constant})$$

$$\int (u+v) \, dx = \int u \, dx + \int v \, dx$$

$$\int uv' \, dx = uv - \int u'v \, dx$$

$$\int_{g(a)}^{g(b)} f(x) \, dx = \int_a^b f(g(t))g'(t) \, dt \quad (\text{substitution rule})$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + c$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + c$$

$$\int \ln x \, dx = x \ln x - x + c$$

$$(\ln x)' = \frac{1}{x}$$

$$(\log_a x)' = \frac{\log_a e}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x = 1/\cos^2 x$$

$$(\cot x)' = -\csc^2 x = -1/\sin^2 x$$

$$(\sec x)' = \sec x \tan x = \sin x/\cos^2 x$$

$$(\csc x)' = -\csc x \cot x = -\cos x/\sin^2 x$$

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$(\tanh x)' = 1/\cosh^2 x$$

$$(\coth x)' = -1/\sinh^2 x$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

$$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{1+x^2}}$$

$$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2-1}}$$

$$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}$$

$$(\operatorname{arcoth} x)' = \frac{1}{1-x^2}$$

**Maclaurin series** (Taylor series with center  $z_0 = 0$ )

$$\cos x = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{(2m)!} = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots$$

$$\sin x = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m+1}}{(2m+1)!} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots$$

$$e^x = \sum_{m=0}^{\infty} \frac{x^m}{m!} = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$$

$$\frac{1}{1-x} = \sum_{m=0}^{\infty} x^m = 1 + x + x^2 + \dots \quad (|x| < 1)$$

$$\ln(1-x) = -\sum_{m=1}^{\infty} \frac{x^m}{m} = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \dots \quad (|x| < 1)$$

$$\arctan x = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m+1}}{2m+1} = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \dots \quad (|x| < 1)$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int \log_a x dx = \frac{x \ln x - x}{\ln a} + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = -\ln|\cos x| + c$$

$$\int \cot x dx = \ln|\sin x| + c$$

$$\int \sec x dx = \ln|\sec x + \tan x| + c$$

$$\int \csc x dx = \ln|\csc x - \cot x| + c$$

$$\int \sinh x dx = \cosh x + c$$

$$\int \cosh x dx = \sinh x + c$$

$$\int \tanh x dx = \ln|\cosh x| + c$$

$$\int \coth x dx = \ln|\sinh x| + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a} + c$$

$$\int \sqrt{a^2-x^2} dx = \frac{1}{2} \left\{ x\sqrt{a^2-x^2} + a^2 \arcsin \frac{x}{a} \right\} + c$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \sqrt{x^2+a^2} dx = \frac{1}{2} \left\{ x\sqrt{x^2+a^2} + a^2 \ln(x+\sqrt{x^2+a^2}) \right\} + c$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \operatorname{arcsinh} \frac{x}{a} + c = \ln(x+\sqrt{x^2+a^2}) + c$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \operatorname{arccosh} \frac{x}{a} + c = \ln(x+\sqrt{x^2-a^2}) + c$$

$$\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + c = \frac{1}{a} \operatorname{arctanh} \frac{x}{a} + c$$

$$\int \sqrt{x^2-a^2} dx = \frac{1}{2} \left\{ x\sqrt{x^2-a^2} - a^2 \ln(x+\sqrt{x^2-a^2}) \right\} + c$$

$$\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c, \quad \int \frac{1}{\sin^2 x} dx = -\frac{1}{\tan x} + c$$

$$\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + c, \quad \int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int \tan^2 x dx = \tan x - x + c, \quad \int \cot^2 x dx = -\cot x - x + c$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx) + c$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx) + c$$

**Growth rate**

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^n} = 0 \quad (n > 0), \quad \lim_{x \rightarrow \infty} \frac{x^n}{a^x} = 0 \quad (a > 1)$$

$$\lim_{x \rightarrow \infty} \frac{a^x}{x!} = 0, \quad \lim_{x \rightarrow \infty} \frac{x!}{x^x} = 0$$

Table of Laplace Transforms

$f(t)$	$F(s)$	$f(t)$	$F(s)$	$f(t)$	$F(s)$
$\delta(t)$	1	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
1, $u(t)$	$\frac{1}{s}$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n$ ( $n = 0, 1, 2, \dots$ )	$\frac{n!}{s^{n+1}}$	$\cos(\omega t + \theta)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$	$\sin(\omega t + \theta)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
$e^{at}$	$\frac{1}{s-a}$	$\cos^2 \omega t$	$\frac{s^2 + 2\omega^2}{s(s^2 + 4\omega^2)}$	$\sin^2 \omega t$	$\frac{2\omega^2}{s(s^2 + 4\omega^2)}$
$e^{at} t^n$ ( $n = 0, 1, 2, \dots$ )	$\frac{n!}{(s-a)^{n+1}}$	$\cosh at$ $= \frac{e^{at} + e^{-at}}{2}$	$\frac{s}{s^2 - a^2}$	$\sinh at$ $= \frac{e^{at} - e^{-at}}{2}$	$\frac{a}{s^2 - a^2}$
$t^a$ ( $a > -1$ )	$\frac{\Gamma(a+1)}{s^{a+1}}$	$\cosh^2 at$	$\frac{s^2 - 2a^2}{s(s^2 - 4a^2)}$	$\sinh^2 at$	$\frac{2a^2}{s(s^2 - 4a^2)}$
$2\sqrt{\frac{t}{\pi}}$	$\frac{1}{s^{3/2}}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t \cos^2 \omega t$	$\frac{s^4 + 2\omega^2 s^2 + 8\omega^4}{s^2(s^2 + 4\omega^2)^2}$	$t \sin^2 \omega t$	$\frac{2\omega^2(3s^2 + 4\omega^2)}{s^2(s^2 + 4\omega^2)^2}$
$\frac{e^{bt} - e^{at}}{2\sqrt{\pi t^3}}$	$\sqrt{s-a} - \sqrt{s-b}$	$t^2 \cos \omega t$	$\frac{2s(s^2 - 3\omega^2)}{(s^2 + \omega^2)^3}$	$t^2 \sin \omega t$	$\frac{2\omega(3s^2 - \omega^2)}{(s^2 + \omega^2)^3}$
$\ln t$	$-\frac{\gamma + \ln s}{s}$	$\frac{2}{t}(1 - \cos \omega t)$	$\ln \frac{s^2 + \omega^2}{s^2}$	$\frac{1}{t} \sin \omega t$	$\arctan \frac{\omega}{s}$
$\frac{1}{t}(e^{bt} - e^{at})$	$\ln \frac{s-a}{s-b}$	$\frac{2}{t}(1 - \cosh at)$	$\ln \frac{s^2 - a^2}{s^2}$	$\frac{1}{t} \sinh at$	$\operatorname{arctanh} \frac{a}{s}$ $\left( = \frac{1}{2} \ln \frac{s+a}{s-a} \right)$
$\frac{k}{2\sqrt{\pi t^3}} e^{-\frac{k^2}{4t}}$ ( $k > 0$ )	$e^{-k\sqrt{s}}$	$ \cos \omega t $	$\frac{s + \omega \operatorname{csch} \frac{\pi s}{2\omega}}{s^2 + \omega^2}$ $= \frac{s(e^{\frac{\pi s}{\omega}} - 1) + 2e^{\frac{\pi s}{2\omega}} \omega}{(e^{\frac{\pi s}{\omega}} - 1)(s^2 + \omega^2)}$	$ \sin \omega t $	$\frac{\omega \operatorname{coth} \frac{\pi s}{2\omega}}{s^2 + \omega^2}$ $= \frac{\omega(e^{\frac{\pi s}{\omega}} + 1)}{(e^{\frac{\pi s}{\omega}} - 1)(s^2 + \omega^2)}$
$\frac{1}{\sqrt{\pi t}} e^{-\frac{k^2}{4t}}$ ( $k > 0$ )	$\frac{e^{-k\sqrt{s}}}{\sqrt{s}}$	$\frac{\cos 2\sqrt{kt}}{\sqrt{\pi t}}$	$\frac{e^{-k/s}}{\sqrt{s}}$	$\frac{\sin 2\sqrt{kt}}{\sqrt{\pi k}}$	$\frac{e^{-k/s}}{s^{3/2}}$
$\frac{2k}{\sqrt{\pi}} e^{-k^2 t^2}$ ( $k > 0$ )	$e^{\frac{s^2}{4k^2}} \operatorname{erfc} \left( \frac{s}{2k} \right)$	$\frac{\cos 2\sqrt{kt}}{2\sqrt{\pi t^3}}$ (*)	$-\sqrt{s} e^{-k/s}$ $-\sqrt{\pi k} \operatorname{erf}(\sqrt{k/s})$	$\frac{\sin 2\sqrt{kt}}{\pi t}$	$\operatorname{erf}(\sqrt{k/s})$
$\frac{1}{k} e^{-\frac{k^2}{4t}}$ ( $k > 0$ )	$\frac{K_1(k\sqrt{s})}{\sqrt{s}}$	$\frac{\cosh 2\sqrt{kt}}{\sqrt{\pi t}}$	$\frac{e^{k/s}}{\sqrt{s}}$	$\frac{\sinh 2\sqrt{kt}}{\sqrt{\pi k}}$	$\frac{e^{k/s}}{s^{3/2}}$
$\frac{1}{2t} e^{-\frac{k^2}{4t}}$ ( $k > 0$ )	$K_0(k\sqrt{s})$	$J_0(at)$	$\frac{1}{\sqrt{s^2 + a^2}}$	$I_0(at)$	$\frac{1}{\sqrt{s^2 - a^2}}$

$\operatorname{erf}(\sqrt{t})$	$\frac{1}{s\sqrt{s+1}}$	$Y_0(at)$	$\frac{-2\operatorname{arcsinh}\frac{s}{a}}{\pi\sqrt{s^2+a^2}}$	$K_0(at)$	$\frac{\operatorname{arccosh}\frac{s}{a}}{\sqrt{s^2-a^2}}$
$\operatorname{erfc}(\sqrt{t})$	$\frac{1}{s+1+\sqrt{s+1}}$ $=\frac{\sqrt{s+1}-1}{s\sqrt{s+1}}$	$J_n(at)$	$\frac{a^n(s+\sqrt{s^2+a^2})^{-n}}{\sqrt{s^2+a^2}}$	$I_n(at)$	$\frac{a^n(s+\sqrt{s^2-a^2})^{-n}}{\sqrt{s^2-a^2}}$
$e^t \operatorname{erf}(\sqrt{t})$	$\frac{1}{\sqrt{s}(s-1)}$	$\frac{1}{t}J_n(at)$ $(n \neq 0)$	$\frac{a^n(s+\sqrt{s^2+a^2})^{-n}}{n}$	$\frac{1}{t}I_n(at)$ $(n \neq 0)$	$\frac{a^n(s+\sqrt{s^2-a^2})^{-n}}{n}$
$e^t \operatorname{erfc}(\sqrt{t})$	$\frac{1}{\sqrt{s}+s}$	$t^n J_n(at)$ $(n > -1/2)$	$\frac{(2a)^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}(s^2+a^2)^{n+\frac{1}{2}}}$	$t^n I_n(at)$ $(n > -1/2)$	$\frac{(2a)^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}(s^2-a^2)^{n+\frac{1}{2}}}$
$\operatorname{erf}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{1-e^{-a\sqrt{s}}}{s}$	$J_0(2\sqrt{kt})$	$\frac{1}{s}e^{-k/s}$	$I_0(2\sqrt{kt})$	$\frac{1}{s}e^{k/s}$
$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{1}{s}e^{-a\sqrt{s}}$	$J_0(a\sqrt{t^2-b^2})$ $(t > b)$	$\frac{e^{-b\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$	$J_0(a\sqrt{t^2+2bt})$	$\frac{e^{b(s-\sqrt{s^2+a^2})}}{\sqrt{s^2+a^2}}$
$\operatorname{Ei}(at)$	$\frac{1}{s} \ln \frac{s+a}{a}$	$\operatorname{Si}(at)$	$\frac{1}{s} \arctan \frac{a}{s}$	$\operatorname{ci}(at)$	$\frac{1}{2s} \ln \frac{s^2+a^2}{a^2}$

**Euler constant**

$$\gamma = \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln n \right) = 0.57721566490153286060651209008240243104215933593992\dots$$

**Gamma function**

$$\Gamma(a) = \int_0^\infty e^{-t} t^{a-1} dt \quad (a > 0), \quad \Gamma(a+1) = a\Gamma(a), \quad \Gamma(k+1) = k! \quad (k = 0, 1, \dots), \quad \Gamma(1/2) = \sqrt{\pi}$$

**Error function and complementary error function**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad \operatorname{erfc}(x) = 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$$

**Sine integral and cosine integral**

$$\operatorname{Si}(x) = \int_0^x \frac{\sin t}{t} dt, \quad \operatorname{si}(x) = \frac{\pi}{2} - \operatorname{Si}(x) = \int_x^\infty \frac{\sin t}{t} dt, \quad \operatorname{ci}(x) = \int_x^\infty \frac{\cos t}{t} dt$$

**Exponential integral and logarithmic integral**

$$\operatorname{Ei}(x) = \int_x^\infty \frac{e^{-t}}{t} dt, \quad \operatorname{li}(x) = \int_0^x \frac{dt}{\ln t}$$

**Fresnel integrals**

$$S(x) = \int_0^x \sin(t^2) dt, \quad C(x) = \int_0^x \cos(t^2) dt$$

**Bessel functions**

Solutions of the differential equation  $x^2 y'' + xy' + (x^2 - \nu^2)y = 0 \quad (\nu \geq 0)$

$$J_\nu(x) = \sum_{m=0}^\infty \frac{(-1)^m x^{2m+\nu}}{2^{2m+\nu} m! \Gamma(\nu+m+1)}, \quad Y_\nu(x) = \frac{1}{\sin \nu\pi} [J_\nu(x) \cos \nu\pi - J_{-\nu}(x)]$$

$$Y_n(x) = \lim_{\nu \rightarrow n} Y_\nu(x) \quad (n = 0, 1, 2, \dots)$$

**Modified Bessel functions**

Solutions of the differential equation  $x^2 y'' + xy' - (x^2 + \nu^2)y = 0 \quad (\nu \geq 0)$

$$I_\nu(x) = i^{-\nu} J_\nu(ix) = \sum_{m=0}^\infty \frac{x^{2m+\nu}}{2^{2m+\nu} m! \Gamma(\nu+m+1)}, \quad K_\nu(x) = \frac{\pi}{2 \sin \nu\pi} [I_{-\nu}(x) - I_\nu(x)]$$

$$K_n(x) = \lim_{\nu \rightarrow n} K_\nu(x) \quad (n = 0, 1, 2, \dots)$$

Table of Inverse Laplace Transforms

$F(s)$	$f(t)$	$F(s)$	$f(t)$	$F(s)$	$f(t)$
1	$\delta(t)$	$e^{as}$	$\delta(t-a)$	$\frac{1}{s^2 + \omega^2}$	$\frac{1}{\omega} \sin \omega t$
$\frac{1}{s}$	1, $u(t)$	$\frac{1}{s-a}$	$e^{at}$	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$
$\frac{1}{s^n}$ ( $n = 1, 2, \dots$ )	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{(s-a)^n}$ ( $n = 1, 2, \dots$ )	$\frac{e^{at} t^{n-1}}{(n-1)!}$	$\frac{1}{s^2 - a^2}$	$\frac{1}{a} \sinh at$
$\frac{1}{s^a}$ ( $a > 0$ )	$\frac{t^{a-1}}{\Gamma(a)}$	$\frac{1}{(s-a)^k}$ ( $k > 0$ )	$\frac{e^{at} t^{k-1}}{\Gamma(k)}$	$\frac{s}{s^2 - a^2}$	$\cosh at$
$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{(s-a)(s-b)}$ ( $a \neq b$ )	$\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)^2 + \omega^2}$	$\frac{1}{\omega} e^{at} \sin \omega t$
$\frac{1}{s^{3/2}}$	$2\sqrt{\frac{t}{\pi}}$	$\frac{s}{(s-a)(s-b)}$ ( $a \neq b$ )	$\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s-a}{(s-a)^2 + \omega^2}$	$e^{at} \cos \omega t$
$\sqrt{s-a} - \sqrt{s-b}$	$\frac{e^{bt} - e^{at}}{2\sqrt{\pi t^3}}$	$\frac{1}{(s^2 + a^2)(s^2 + b^2)}$ ( $a \neq b$ )	$\frac{b \sin at - a \sin bt}{ab(b^2 - a^2)}$	$\frac{1}{s(s^2 + \omega^2)}$	$\frac{1}{\omega^2} (1 - \cos \omega t)$
$\frac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{\frac{-(a+b)t}{2}} \times I_0\left(\frac{a-b}{2}t\right)$	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$ ( $a \neq b$ )	$\frac{\cos at - \cos bt}{b^2 - a^2}$	$\frac{1}{s^2(s^2 + \omega^2)}$	$\frac{1}{\omega^3} (\omega t - \sin \omega t)$
$\frac{1}{\sqrt{s^2 + a^2}}$	$J_0(at)$	$\frac{1}{s^4 + 4k^4}$	$\frac{1}{4k^3} \begin{pmatrix} \sin kt \cosh kt - \\ \cos kt \sinh kt \end{pmatrix}$	$\frac{1}{(s^2 + \omega^2)^2}$	$\frac{\sin \omega t - \omega t \cos \omega t}{2\omega^3}$
$\frac{s}{(s-a)^{3/2}}$	$\frac{e^{at}(1+2at)}{\sqrt{\pi t}}$	$\frac{s}{s^4 + 4k^4}$	$\frac{1}{2k^2} \sin kt \sinh kt$	$\frac{s}{(s^2 + \omega^2)^2}$	$\frac{t \sin \omega t}{2\omega}$
$\frac{1}{\sqrt{s^2 - a^2}}$	$I_0(at)$	$\frac{1}{s^4 - k^4}$	$\frac{1}{2k^3} (\sinh kt - \sin kt)$	$\frac{s^2}{(s^2 + \omega^2)^2}$	$\frac{\sin \omega t + \omega t \cos \omega t}{2\omega}$
$\frac{1}{(s^2 - a^2)^k}$ ( $k > 0$ )	$\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a}\right)^{k-\frac{1}{2}} \times I_{k-1/2}(at)$	$\frac{s}{s^4 - k^4}$	$\frac{1}{2k^2} (\cosh kt - \cos kt)$	$\frac{s^3}{(s^2 + \omega^2)^2}$	$\cos \omega t - \frac{1}{2} \omega t \sin \omega t$
$\frac{1}{s} e^{-k/s}$	$J_0(2\sqrt{kt})$	$\frac{1}{s} e^{k/s}$	$I_0(2\sqrt{kt})$	$\frac{\ln s}{s}$	$-\ln t - \gamma$
$\frac{e^{-k/s}}{\sqrt{s}}$	$\frac{\cos 2\sqrt{kt}}{\sqrt{\pi t}}$	$\frac{e^{k/s}}{\sqrt{s}}$	$\frac{\cosh 2\sqrt{kt}}{\sqrt{\pi t}}$	$\ln \frac{s-a}{s-b}$	$\frac{1}{t} (e^{bt} - e^{at})$
$\frac{e^{-k/s}}{s^{3/2}}$	$\frac{\sin 2\sqrt{kt}}{\sqrt{\pi k}}$	$\frac{e^{k/s}}{s^{3/2}}$	$\frac{\sinh 2\sqrt{kt}}{\sqrt{\pi k}}$	$\ln \frac{s^2 + \omega^2}{s^2}$	$\frac{2}{t} (1 - \cos \omega t)$
$e^{-k\sqrt{s}}$ ( $k > 0$ )	$\frac{k}{2\sqrt{\pi t^3}} e^{-\frac{k^2}{4t}}$	$\arctan \frac{\omega}{s}$	$\frac{1}{t} \sin \omega t$	$\ln \frac{s^2 - a^2}{s^2}$	$\frac{2}{t} (1 - \cosh at)$
$\frac{e^{-k\sqrt{s}}}{\sqrt{s}}$ ( $k > 0$ )	$\frac{1}{\sqrt{\pi t}} e^{-\frac{k^2}{4t}}$	$\frac{1}{s} \arctan \frac{\omega}{s}$	$\text{Si}(\omega t)$	$\ln \frac{s^2 + \omega^2}{s^2 - a^2}$	$\frac{2}{t} (\cosh at - \cos \omega t)$