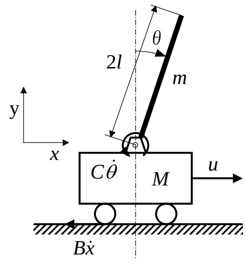
# 倒立振子の制御

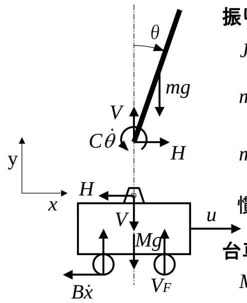
力 $u$ [N] を操作して，
台車の位置　$x = 0$ [m] ,
振り子の傾き $\theta = 0$ [rad] で
静止させたい




**振り子**（均質な棒）
長さ $2l$ [m],　質量 $m$ [kg]
軸の粘性摩擦係数 $C$ [kg m$^2$/s]

**台車**
質量 $M$ [kg]
粘性摩擦係数 $B$ [kg/s]

# 運動方程式 (Newton-Euler)




**振り子**

$$J\ddot{\theta} = Vl\sin\theta - Hl\cos\theta - C\dot{\theta} \quad (1\text{-}1)$$

$$m\frac{d^2}{dt^2}\left(x + l\sin\theta\right) = H \quad (1\text{-}2)$$

$$m\frac{d^2}{dt^2}\left(l\cos\theta\right) = V - mg \quad (1\text{-}3)$$

**慣性モーメント**　$J = ml^2/3$

**台車**

$$M\ddot{x} = u - B\dot{x} - H \quad (1\text{-}4)$$

# 運動方程式　その1

式(1-1)～(1-4) から，$H$ と $V$ を消去する

$$\left(M+m\right)\ddot{x} + ml\cos\theta\cdot\ddot{\theta} - ml\dot{\theta}^2\sin\theta + B\dot{x} = u$$

$$ml\cos\theta\cdot\ddot{x} + \left(J + ml^2\right)\ddot{\theta} - mlg\sin\theta + C\dot{\theta} = 0$$

ただし　$J = ml^2/3$

$$\begin{bmatrix} M+m & ml\cos\theta \\ ml\cos\theta & J+ml^2 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} -ml\dot{\theta}^2\sin\theta \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -mlg\sin\theta \end{bmatrix} + \begin{bmatrix} B\dot{x} \\ C\dot{\theta} \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}$$

$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{d}(\dot{\boldsymbol{q}}) = \boldsymbol{\tau}$　の形

# 運動方程式　その2

$\ddot{x}$ と $\ddot{\theta}$ について解くと

$$\begin{aligned}\ddot{x} = \{&\left(J+ml^2\right)\cdot u - B\left(J+ml^2\right)\cdot\dot{x} \\ &+ Cml\cos\theta\cdot\dot{\theta} + \left(J+ml^2\right)ml\sin\theta\cdot\dot{\theta}^2 \\ &- m^2l^2g\sin\theta\cos\theta\}/\alpha \end{aligned} \quad (2\text{-}1)$$

$$\begin{aligned}\ddot{\theta} = \{&-ml\cos\theta\cdot u + Bml\cos\theta\cdot\dot{x} \\ &- C\left(m+M\right)\cdot\dot{\theta} - m^2l^2\sin\theta\cos\theta\cdot\dot{\theta}^2 \\ &+ \left(m+M\right)mlg\sin\theta\}/\alpha \end{aligned} \quad (2\text{-}2)$$

ただし

$$\alpha = J(m+M) + Mml^2 + m^2l^2\sin^2\theta$$

# 運動方程式 (Lagrange)

| | | |
|---|---|---|
| 運動エネルギー | $K = \left(M\dot{x}^2 + J\dot{\theta}^2 + m\dot{X}^2 + m\dot{Y}^2\right)/2$ | (3-1) |
| ただし， | $X = x + l\sin\theta,\quad Y = l\cos\theta$ | |
| 位置エネルギー | $U = mgl\cos\theta$ | (3-2) |
| ラグランジュ関数 | $L = K - U$ | (3-3) |
| 損失エネルギー | $D = \left(B\dot{x}^2 + C\dot{\theta}^2\right)/2$ | (3-4) |
| **運動方程式** | $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}}\right) - \dfrac{\partial L}{\partial x} + \dfrac{\partial D}{\partial \dot{x}} = u$ | (3-5) |
| | $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\theta}}\right) - \dfrac{\partial L}{\partial \theta} + \dfrac{\partial D}{\partial \dot{\theta}} = 0$ | (3-6) |

式(3-1)～(3-6) を整理すると，式(2-1)，(2-2) となる

# 状態方程式

**状態変数ベクトル**

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} x & \theta & \dot{x} & \dot{\theta} \end{bmatrix}^T \quad (4\text{-}1)$$

**状態方程式**

$$\dot{\boldsymbol{x}} = \begin{bmatrix} x_3 \\ x_4 \\ \left(a_{32}\sin x_2\cos x_2 + a_{33}x_3 + a_{34}x_4\cos x_2 + a_{35}x_4^2\sin x_2 + b_3u\right)/\left(1+\alpha\sin^2 x_2\right) \\ \left(a_{42}\sin x_2 + a_{43}x_3\cos x_2 + a_{44}x_4 + a_{45}x_4^2\sin x_2\cos x_2 + b_4u\cos x_2\right)/\left(1+\alpha\sin^2 x_2\right) \end{bmatrix}$$

ただし，　(4-2)

$\alpha_0 = J\left(m+M\right) + Mml^2$,　$\alpha = m^2l^2/\alpha_0$,　$J = ml^2/3$
$a_{32} = -m^2l^2g/\alpha_0$,　$a_{33} = -B\left(J+ml^2\right)/\alpha_0$,　$a_{34} = Cml/\alpha_0$,　$a_{35} = \left(J+ml^2\right)ml/\alpha_0$
$a_{42} = \left(m+M\right)mlg/\alpha_0$,　$a_{43} = Bml/\alpha_0$,　$a_{44} = -C\left(m+M\right)/\alpha_0$,　$a_{45} = -m^2l^2/\alpha_0$
$b_3 = \left(J+ml^2\right)/\alpha_0$,　$b_4 = -ml/\alpha_0$